# ComputerOriented Texts <br> <br> \section*{from <br> <br> \section*{from Prentice PrenticeHall} 

Hall}}

PROGRAMMING LANGUAGE/ONE by Frank Bates, Union Carbide Corporation and Mary L. Douglas, The Johns Hopkins University. Provides a broadbased introduction to $\mathrm{PL} / 1$ by presenting common data processing principles and their implementation in PL/1. Covers PL/1 in sufficient detail for most data processing situations and presents this language in the context of general computer concepts, giving the beginner an appreciation of data processing by computers. March 1967, 384 pp., $\$ 5.95$

COMPUTER CALCULATIONS FOR MULTICOMPONENT VAPOR-LIQUID EQUILIBRIA by J. M. Prausnitz, C. A. Eckert, R. V. Orye and J. P. O'Connell, all of the University of California, Berkeley. Presents a modern and rational method of estim. ating equilibria as required in the design of distillation and related separation equipment.Techniques presented are semi-empirical, but have a sound basis in molecular thermodynamics. Provides detailed computer programs with worked-out examples. Requires only limited knowledge of binary and pure-component data. January 1967, 256 pp., $\$ 9.95$

UNSTEADY STATE PROCESSES WITH APPLICATIONS IN MULTICOMPONENT DISTILLATION by Charles D. Holland, Texas A. \& M. University. Covers the setting up and computer solution of equations which describe the fundamental processes -- as mass transfer - that occur within a distillation column. Treats the dynamic operation of both continuous and batch distillation columns in the process of separating multicomponent mixtures, with consideration of the problems in fluid dynamics, heat transfer, mass transfer, mixing, unsteady state material, energy balance and process control. 1966, 384 pp., $\$ 15.95$

SIMULATION USING DIGITAL COMPUTERS by George W. Evans II and Graham F. Wallace, both of Stanford Research Institute, and Georgia L. Sutherland, Stanford University. Treats the creation and use of computer simulation models and routines for the study of systems. Covers systematizing and generalizing in order to prepare for a mathematical and programming theory of simulation. May 1967, 224 pp., $\$ 8.25$

DESIGN OF REAL-TIME COMPUTER SYSTEMS by James T. Martin, Systems Research Institute, International Business Machines Corporation. Describes the basic decisions that must be made by management and systems analysts in considering and planning the use of real-time data processing, discussing the calculations required and the hardware and techniques available. Covers techniques used in on-line and real-time systems, applications of real-time techniques, hardware features, design calculations and simulation, implementation problems, and the use of terminals, communication lines and random-access files, among other subjects. August 1967, 564 pp., $\$ 12.95$
for approval copies, write: Box 903


## BOOKS FROM BLAISDELL

Series on
Numerical Analysis and Computer Science
The first three titles are:
NUMERICAL INTEGRATION
Philip J. Davis, Brown University
Philip Rabinowitz, Weizmann Institute, Israel
1966 • 255 pages • $\$ 7.50$
STATISTICAL COMPUTATIONS ON A DIGITAL COMPUTER
William J. Hemmerle, University of Rhode Island
1967 - 230 pages • $\$ 7.50$

NUMERICAL METHODS FOR TWO POINT BOUNDARY VALUE PROBLEMS
Herbert B. Keller, California Institute of Technology
In press

Series on
Solid Mechanics
The first four titles are:
CREEP IN ENGINEERING STRUCTURES
Jan A. H. Hult, Chalmers University of Technology, Gothenburg

VISCOELASTICITY
Wilhelm Flügge, Stanford University

PRINCIPLES OF STRUCTURAL STABILITY
Hans Ziegler, Federal Institute of Technology, Zurich

## THERMOELASTICITY

Heinz Parkus, Institute of Technology, Vienna
In press
An Additional Title of Interest
PROBLEMS IN GROUP THEORY
John D. Dixon, University of New South Wales

A Division of Ginn and Company 275 Wyman Street, Waltham, Massachusetts 021.54

## MICROFICHE

Beginning with this issue of MATHEMATICS OF COMPUTATION, we may from time to time include auxiliary material on an attached "microfiche" card. This card which complies with the American Standards Association standards for archival storage, may contain complete tables reviewed in the journal, tables contained in the repository of unpublished mathematical tables and/or an extension of remarks contained in research articles. We offer this service so that information which cannot be included in the journal itself, due to lack of space, may be made available to subscribers. Microfiche reading equipment is available in many libraries and also several manufacturers offer inexpensive portable readers.
The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

# Mathematics of Computation 

TABLE OF CONTENTS

July 1967
On the Construction of Discrete Approximations to Linear Differential Expressions C. Ballester \& V. Pereyra

297

Parallel Methods for the Numerical Integration of Ordinary Differential Equations
Willard L. Miranker \& Werner Liniger ..... 303
A Fundamental Solution for a Biharmonic Finite-Difference Operator .R. Bruce Simpson ..... 321
Numerical Solution of the Minimal Surface Equation Paul Concus ..... 340
Split Operator Methods for Hyperbolic Systems in $p$ Space Variables
A. R. Gourlay \& A. R. Mitchell ..... 351
A Steady State Phase Change Problem. ..... 355
On the Relative Efficiencies of Gradient Methods John Greenstadt ..... 360
Quasi-Newton Methods and their Application to Function Minimisation C. G. Broyden ..... 368
Quadratures with Remainders of Minimum Norm. II . R.E. Barnhill \& J. A. Wixom ..... 382
A Modified Monte-Carlo Quadrature. II . . . . . . . . . . . . . . Seymour Haber ..... 388
Truncation Errors in Padé Approximations to Certain Functions: An Alternative Approach David Elliott ..... 398
Calculations of Integrals of Products of Bessel Functions
J. E. Kilpatrick, Shigetoshi Katsura \& Yuji Inoue ..... 407
Bessel-Function Integrals Needed for Two Classes of Physical ProblemsCharles E. McQueary \& Lawrence R. Mack413
Rational Approximations to the Incomplete Elliptic Integrals of the First and Second Kinds
Wyman G. Fair \& Yudell L. Luke ..... 418
Some Limiting Values and Two Error Estimation Procedures for Power Series Approxima- tions. ..... 423
Eigenvalues and Eigenvectors of Hilbert Matrices of Order 3 Through 10
Henry E. Fettis \& James C. Caslin ..... 431
A Note on the Class Number of Imaginary Quadratic Fields . . . .Raymond G. Ayoub ..... 442
A Survey of Equal Sums of Like Powers L. J. Lander, T. R. Parkin \& J. L. Selfridge ..... 446
On the Sum of the Squares of Two Consecutive Integers. Edward L. Cohen ..... 460
On the Distribution of Parity in the Partition Function
Thomas R. Parkin \& Daniel Shanks 46
Technical Notes and Short PapersNote on Partitions Modulo 5.Morris Newman481
On First Appearance of Prime Differences L. J. Lander \& T. R. Parkin ..... 483
Consecutive Primes in Arithmetic Progression. L. J. Lander \& T. R. Parkin ..... 489
Convergence of Successive Substitution Starting Procedures . . A. C. R. Newbery ..... 489
A Polynomial Approximation Converging in a Lens-Shaped Region Jay A. Leavirt ..... 491
Chebyshev Expansions for Integrals of the Error Function .Van E. Wood ..... 494
An Integral Representation for the Modified Bessel Function of the Third Kind, Com- putable for Large, Imaginary Order . . . .James D. Lear \& James E. Sturm ..... 496
Mixed Algebraic-Exponential Interpolation Using Finite Differences J. W. Layman ..... 498
Reviews and Descriptions of Tables and Books ..... 501
Schütte 56, Ondrejka 57, Cater 58, White 59, Alexander \& Vok 60, Ireson 61,Radanović 62, Keilson 63, Fettis \& Caslin 64, Fettis \& Caslin 65, Zhurina \&Karmazina 66, Clenshaw \& Picken 67, Krugliák \& Whitman 68, Barnhill \&Wixom 69, Alder, Fernbach \& Rotenberg 70, Smythe \& Johnson 71, Visapää 72 ,Conte 73, Garabedian 74, Johnson 75, Isaacson \& Keller 76, Ballester \& Pereyra77, Berkeley \& Bobrow 78, Ekman \& Fröberg 79, Andree 80, Greenberger, Jones,Morris \& Ness 81, Opler 82, Farina 83
Table Errata ..... 523
Abramowitz \& Stegun 411, Ḣenderson 412, Magnus \& Oberhettinger 413, Magnus \& Oberhettinger 414
CorrigendumCody \& Thacher.525
Notes
New Journals ..... 525

